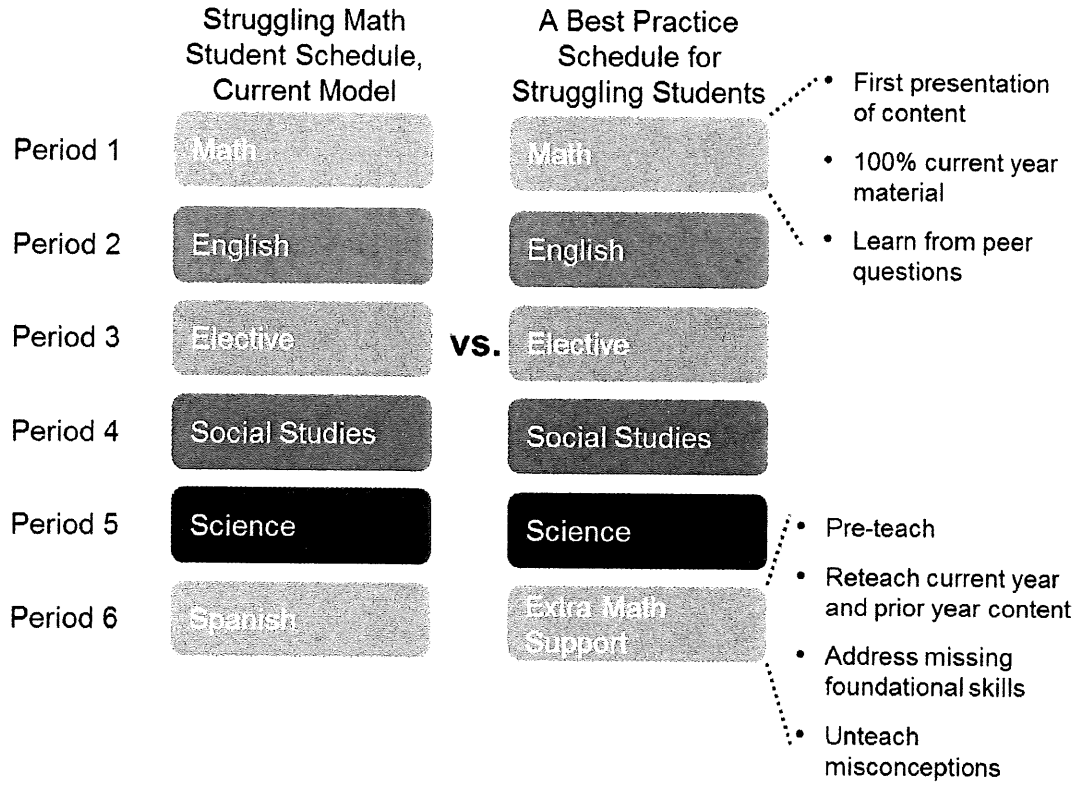
Providing extra time to pre-teach materials, re-teach the day's lesson, address missing foundational skills, and un-teach misconceptions is a best practice to supporting all students struggling in math and ELA at the secondary schools.

There is not a consistent practice of providing additional instructional time in either ELA or math at the high school level. Interviews indicated that there are no district wide formalized or systematized interventions for math or ELA content at the high school level.

At the middle school level, many schools offer "seminars" in math and ELA, however their use and purpose is not consistent across the district. Interviews indicated that these seminar courses are typically not structured to fill in learning gaps and build skills that the student might have missed in previous years, but they are primarily a repeat of the content from the student's core math or ELA class.

Schedules for Struggling Students in the Current and Best Practice Models

Current vs. Best Practice Struggling Student Schedule



Similar to elementary reading, there are not clear benchmarks or a systematized approach to identify struggling students at both the middle and high school levels in ELA and math. Rather, schools rely on teacher-written assessments and teacher recommendations, sometimes from the previous year, to identify students who are falling behind.

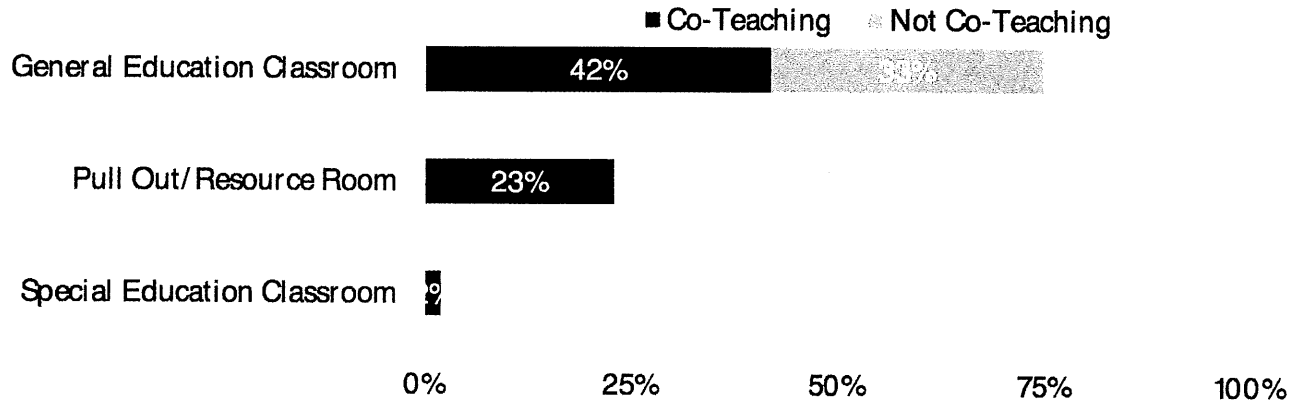
1c. Implications for current practices

In Howard County Public Schools, support for students with IEPs often occurs in the form of "increased adult intensity," rather than extended time; in this model, students are assigned multiple adults to support them at the same time (e.g., collaboration, co teaching, 1-to-1 support), instead of getting extended time on task. Shifting away from a high-intensity model of student support can maximize student learning and free up funds to support other opportunities to raise achievement.

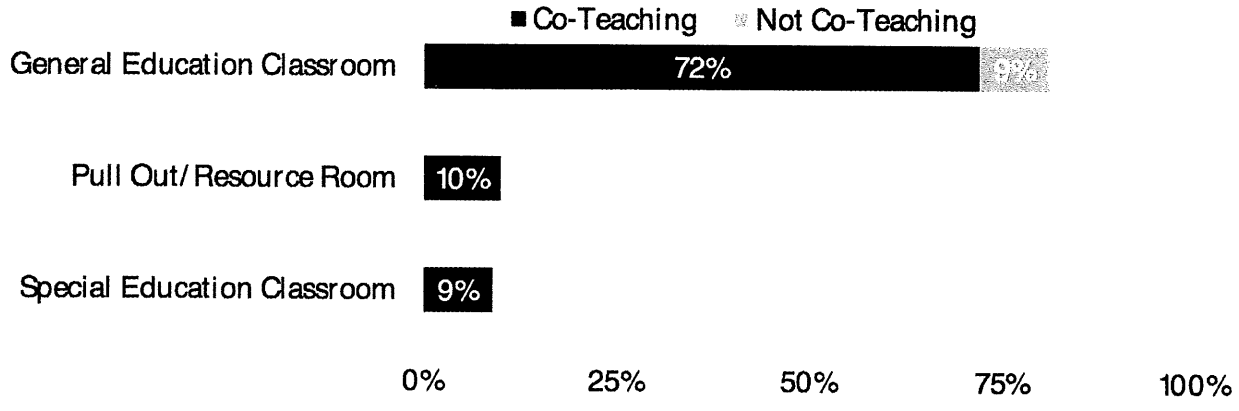
In Howard County Public Schools, "collaboration," two adults at once, is the most common form of special education service delivery:

- Approximately 75% of elementary inclusion special educators' direct service time is spent either in a co-teaching or push-in setting.
- Similarly, 81% of secondary inclusion special education teachers' direct service time is in a setting with a general educator present.

Percentage of Direct Service Time Spent by Inclusion Special Education Teachers
Elementary Level



Percentage of Direct Service Time Spent by Inclusion Special Education Teachers
Secondary Level



The district has placed co-teaching at the forefront of its efforts to help struggling students with special needs. National research, however, suggests that co-teaching seldom raises student achievement. In his 2009 review of educational research, John Hattie notes that no studies have shown student gains from co-teaching and that on average it actually produced less or equal learning than a class with a single teacher, while costing twice as much. This is because while co-teaching represents higher "intensity" of support (i.e., multiple adults providing support at the same time), it does not mean extended time on task with a content-strong teacher for the struggling student.

Interviews with teachers across the country who co-teach suggest that co-teaching, while promising in theory, is often executed poorly. Effective co-teaching requires a high level of collaborative planning between the general education and special education teachers, which requires daily common planning time. Teachers often express not having sufficient time to meet and plan lessons in their teams. Lack of planning results in lack of consistency in the pair's instruction of content, as the two teachers may have different goals for the students. Providing common planning time, however, typically increases staffing requirements by 20% or more.

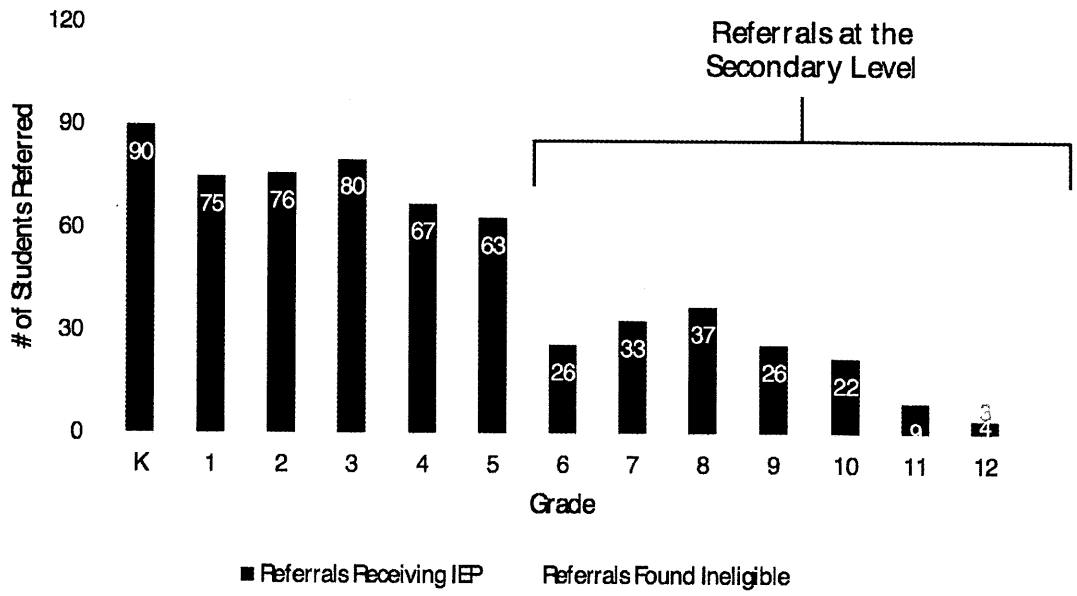
Similar challenges exist in Howard County. Interviews suggested that structures for common planning do not exist consistently across the district, which often renders the co-teaching model frustrating and ineffective. Many staff acknowledged that co-teaching was not being implemented with fidelity due to limited co-planning time and other demands that frequently pull either the general education teacher or the special education teacher out of the classroom. For instance, during DMC's classroom visits, three co-taught classrooms were observed to be missing either the special education teacher, the general education teacher, or both.

The need for more general education interventions

The IEP and 504 referral rates, especially at the secondary level—in addition to the achievement gaps—in Howard County Public Schools support the premise that current general education interventions need to be expanded.

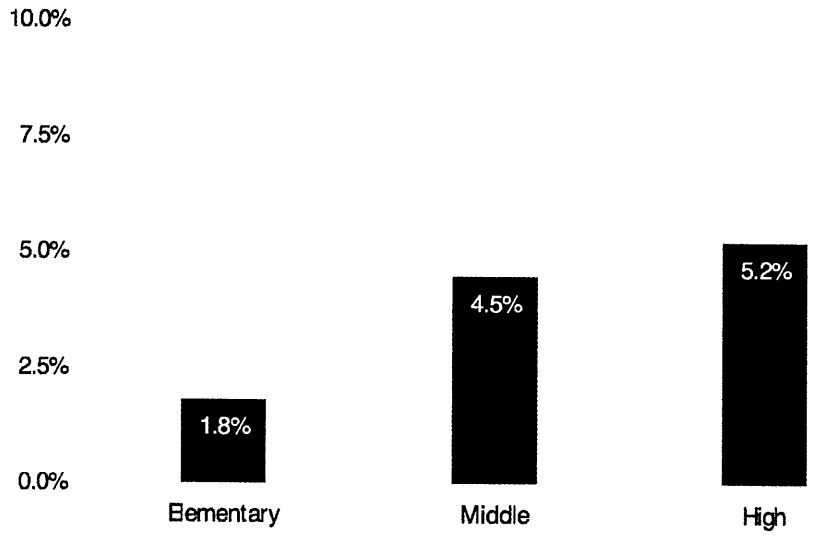
In most districts referrals to special education or requests for 504s peak around 3rd or 4th grade and diminish to nearly zero by the start of middle school. This is not the case in the district.

Students' Initial Referral to Special Education
By Grade Level



- The district refers a significant amount of students for IEPs at the secondary level.
- Only 46% of those students are found eligible for an IEP.

Percentage of Total Enrollment Receiving 504 Services
By Grade Level



- The rate at which students receive 504 services in high school is nearly three times the rate in elementary school.
- The rate at the high school is more than 4 times the national average.

70 FRANKLIN STREET • BOSTON MA 02110 • 877-DMC-3500 • WWW.DMCOUNCIL.ORG

The IEP referral rate and the 504 identification rate both indicate that students in the district need greater support as they move into the secondary level, but the current model within general education does not provide this.

2. Ensure that students who struggle receive instruction from instructors with subject-specific training during core classes and interventions.

Extra time on task is not sufficient for struggling students to master grade level material. The training and knowledge of the teacher also matters a great deal.

2a. Elementary Reading

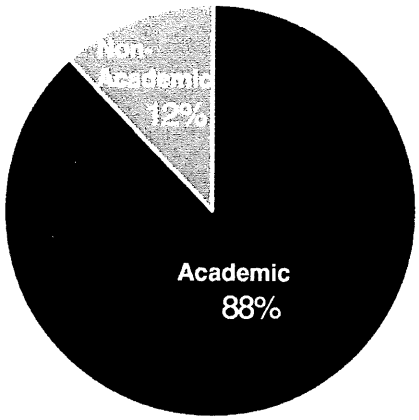
For students who struggle, research indicates that the subject-specific training of the instructor has significant bearing on the student's likelihood of achieving grade level mastery. Effective teachers of reading have extensive training in the teaching of reading. Often, special educators have deep expertise in pedagogy but limited background in the teaching of reading. Districts that have made the most significant gains among struggling readers have done so by providing teachers skilled in the teaching of reading extra time with struggling students.

Paraprofessionals can play an important role in supporting many students with special needs, especially for behavioral and physical support; however, the overuse of paraprofessional support can often limit students' learning and independence, in addition to making the job frustrating for paraprofessionals. When students struggle in reading, it is generally more beneficial for their learning to spend extra time with teachers or interventionists highly skilled in the teaching of reading than with paraprofessionals, who generally do not have extensive training in the teaching of reading.

Interviews, classroom visits, and data from the schedule sharing all indicate that the background and training of staff providing elementary reading instruction varied significantly across the district. Staff in the focus groups explained that reading specialists or special education teachers could both lead reading instruction lessons, and paraprofessionals could provide reading instruction if the materials were prepared by a special education teacher. There is not a consistent approach across the district, and during classroom visits DMC observed all three practices being implemented.

As the data from the schedule sharing illustrates, special education teachers, paraprofessionals, and student assistants are all providing a significant amount of core academic instruction in Howard County Public Schools.

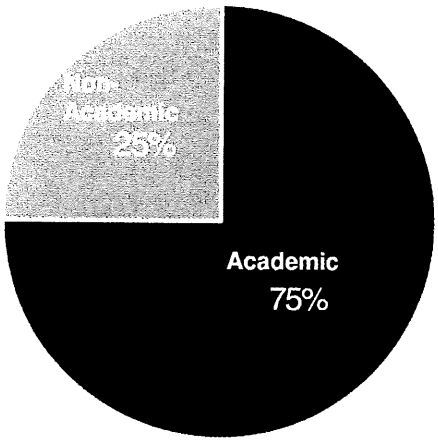
Special Education Teachers (Inclusion) Instructional Topic (141.0 FTE)[1]
Elementary Level Only



Academic topic	% time spent
Reading	39%
Math	31%
Writing	10%
Science	4%
Social Studies	4%
Total academic instruction	88%

- Special education teachers are spending nearly all of their direct service time providing content instruction, including 39% of that time on reading instruction.

Special Education Paraeducators (Inclusion) Instructional Topic (110.0 FTE)*
Elementary Level Only

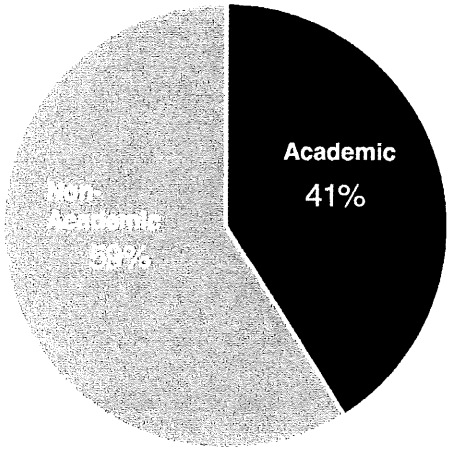


Academic topic	% time spent
Reading	24%
Math	23%
Writing	12%
Social Studies	8%
Science	8%
Total academic instruction	75%

- Special education paraeducators are spending nearly three out of four hours of their time spent with students providing content instruction or support, including nearly a quarter of their time on both reading and on math.
- General education paraprofessionals did not share their schedules, but the district has many such staff, many involved in reading instruction.

[1] Academic and non-academic support is equal to 100% of student support (direct service) time.
* This study only collected data on special education and early childhood paraprofessional staff. General education paraprofessional staff are not included in this analysis.

Student Assistants (Inclusion) Instructional Topic (35.0 FTE)
Elementary Level Only



Academic topic	% time spent
Math	16%
Reading	11%
Writing	6%
Social Studies	4%
Science	4%
Total academic instruction	41%

- Student assistants spend significantly less of their direct service time on core instruction than paraprofessionals, but they are still spending two out of five hours of that time providing content instruction or support.

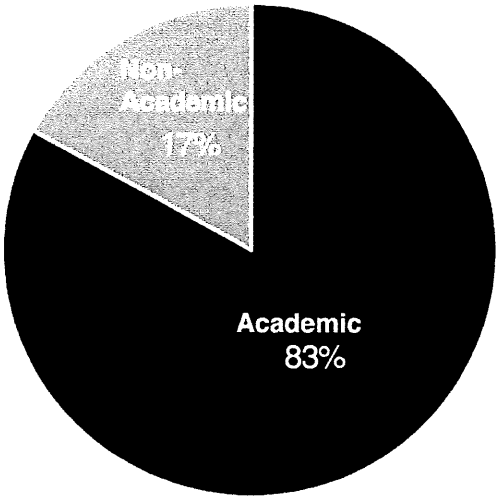
2b. Secondary ELA and Math

Just as the skill and training of the instructor is vital for the reading success of students at the elementary schools, this is just as true in secondary math and English. Typically, a teacher who has engaged in extensive training and study of a subject is more likely to have intricate working knowledge of the subject and an ability to understand and explain the content to a struggling student in a way that will lead to mastery. For students with or without IEPs who struggle at the secondary level, research shows the content expertise of the instructor has significant bearing on the student's likelihood of mastering the grade level material.

Similar to the reading support at the elementary level, there is a wide variance in what types of staff are providing intervention and support for secondary ELA and math. Both math and ELA instruction can be provided to struggling students with or without an IEP in a variety of settings and by instructors with a variety of backgrounds including in a co-taught classroom, by a reading specialist or a special education teacher, or by a special education teacher and a paraeducator, among other combinations.

Special Education Teachers (Inclusion) Instructional Topic (231.0 FTE)

Secondary Level Only

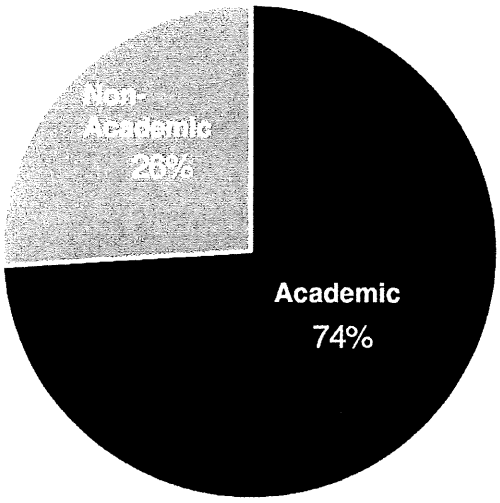


Academic topic	MS	HS
Math	33%	22%
Writing	30%	12%
Reading	19%	17%
Science	6%	15%
Social Studies	5%	8%
Total academic instruction	92%	74%

- Special education teachers are spending most of their direct service time providing content instruction, including 33% of that time on math and 49% on ELA instruction at the middle school level.

Special Education Paraeducators (Inclusion) Instructional Topic (102.0 FTE)

Secondary Level Only



Academic topic	MS	HS
Social Studies	26%	15%
Science	25%	17%
Math	16%	13%
Reading	10%	12%
Writing	5%	9%
Total academic instruction	82%	66%

- Special education paraeducators are spending nearly three out of four hours of their time spent with students providing content instruction or support, much of it in math and social studies. In many districts support is not provided in these subjects.

2c. Implications for current model

Co-teaching does not provide students with full access to a teacher with subject-specific training at the elementary or the secondary level. Although struggling students might be in the same classroom as a teacher with deep content knowledge, special education teachers and paraeducators are also providing a significant amount of their instruction.

Interviews also indicated that within the current co-teaching model some educators, including special education teachers and paraeducators, are lowering expectations for students with disabilities. Focus group participants shared that it is not uncommon for tests and curriculum to be modified, which often "takes the rigor out" of the assignment. For instance, multiple staff members mentioned that assessments are often cut in half for students with disabilities in co-taught classrooms. Providing students with full access to instructors with deep understanding of the standards and content will help combat the lowering of expectations for students with disabilities.

Increasing the role of general education staff is consistent with the district's commitment to inclusion. It would also serve the many students who are being referred to special education or getting 504s at the secondary level. More students can be helped in a more impactful way at no added cost.

3. Consider shifting the roles of paraprofessional staff to emphasize providing nonacademic support, rather than content instruction, for students with mild to moderate disabilities.

The district has invested significantly in providing paraprofessional support for students with disabilities. Interviews indicated that there are three main types of paraprofessional staff that provide services to students in the district, totaling more than 700 FTE: special education paraeducators, student assistants, and temporary employees. This study collected extensive data on paraeducators and student assistants. The distinctions between each position are explained below, although each role performs similar functions overall:

- Special education paraeducators: provide support and sometimes instruction to small groups of students with disabilities
- Student assistants: provide predominantly 1-to-1, non-academic support to students, although it is not uncommon for them to provide instruction as well
- Temporary employees: contracted through an outside agency, primarily help to ensure that students do not act out in class and often are staffed 1-to-1

The district also has a large number of general education paraprofessionals.

Two factors could be contributing to the significant investment in paraprofessional support. First, the district relies on paraprofessional staff to provide a significant amount of instruction to struggling students. Second, the artificial stratification of paraprofessional staff into three distinct roles could be contributing to the higher-than-typical staffing levels. Interviews indicated that oftentimes the roles and responsibilities of the three different paraprofessional positions were ambiguous or overlapping, which could cause multiple staff to be assigned to similar activities.

4. Consider increasing the amount of time related service providers spend with students, while also closely managing group size through thoughtful scheduling.

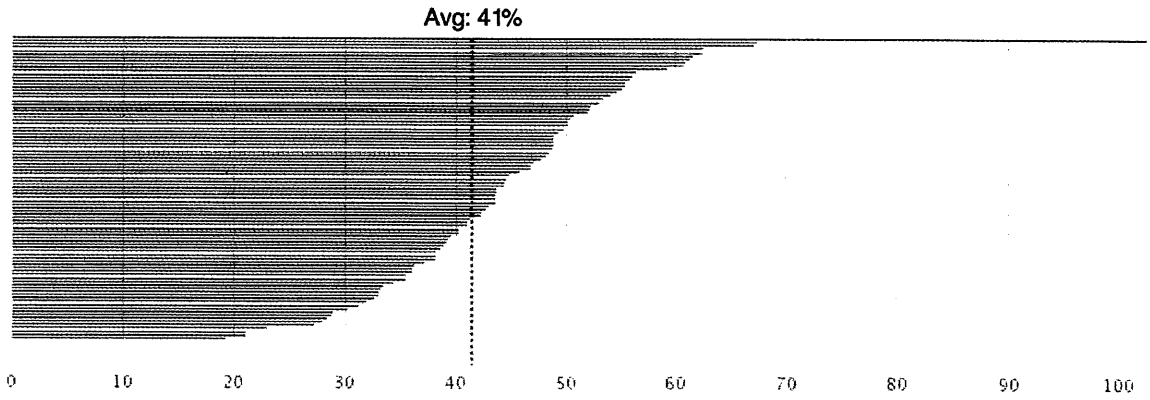
Taking a proactive role in managing related services could allow the district to free up funds to service more students without reducing a minute of service to students.

4a. Speech and Language Pathologists

Speech and language pathologists are an important component of many student's IEPs. They spend time working directly with students, while also participating in evaluations, report writing, and data analysis.

Speech and Language Pathologist Direct Student Support (105.5 FTE)

Direct service is calculated based on the percent of time spent with students in the contracted work week.



- On average, speech therapists spend 41% of the contracted work week serving students.

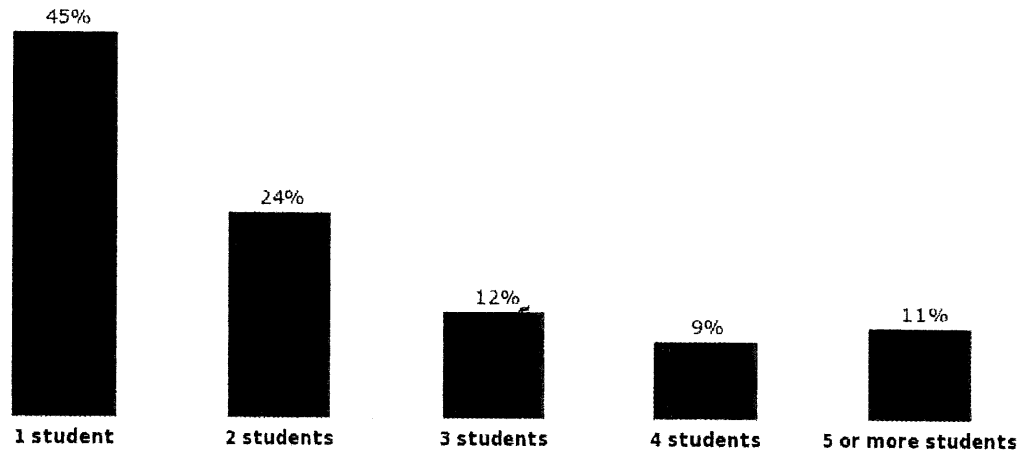
Speech and Language Pathologist Activities (105.5 FTE)

Activity	% time spent
Therapy with students	41%
Total direct service	**41%**

Paperwork/ IEP/ IFSP writing	10%
Planning/ materials preparation	9%
Collaboration with colleagues (email, phone, in-person)	7%
Attend meeting (IEP/IFSP)	5%
Personal lunch	5%
Attend meeting (other than IEP/IFSP)	5%
Professional development	5%
Assessing/ observing students	3%
Travel	3%
Medicaid billing/ service documentation	3%
Parent counseling/ training	2%
Assigned school duties (i.e. bus duty, lunch duty, etc.)	2%
IEP/ IFSP testing/ assessment	2%
Over reported	-2%
Total Indirect Service	**59%**

The average speech therapist in the district serves 28 students. Nationally the typical caseload is over 50 students. The low caseload is a consequence of much time in meetings and doing paperwork, coupled with providing much service 1:1. Speech therpaists provide nearly half of their services 1:1.

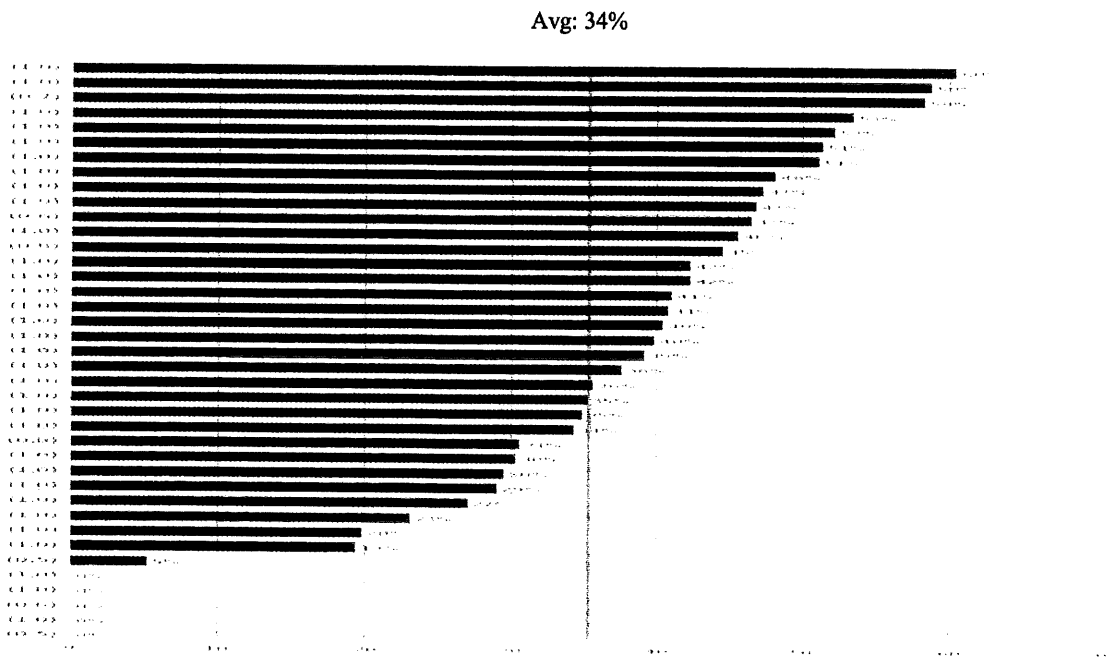
Speech and Language Pathologist Group Size (105.5 FTE)



4b. Occupational Therapists

Much like speech and language pathologists, occupational therapists provide very important services to students with disabilities, but also have other responsibilities.

Occupational Therapist Direct Student Support (40.2 FTE)



- Occupational therapists, on average, provide less than two days per week of services to students.
- The range of direct service time is quite large, with five individual practitioners indicating that they spent no time with students and one practitioner spending 60% of time with students.

4c. Physical Therapists

Physical therapists have quite similar schedules to those of occupational therapists and speech and language pathologists. The 12.4 FTE of physical therapists in the district provide an average of 35% direct service to students, with practitioners ranging from below 20% to above 50%. They also spend 26% of their time doing IEP paperwork or traveling.

5. Consider expanding the roles and responsibilities of school and central office-based administrators to more closely manage how special education staff use their time.

As the district moves to best practice service delivery models, district leaders can set explicit expectations for how services are provided, how much time in a day staff provide instruction and how many students are helped at once by a teacher. These service delivery, workload and group size guidelines are very common in general education, but less so in special education.

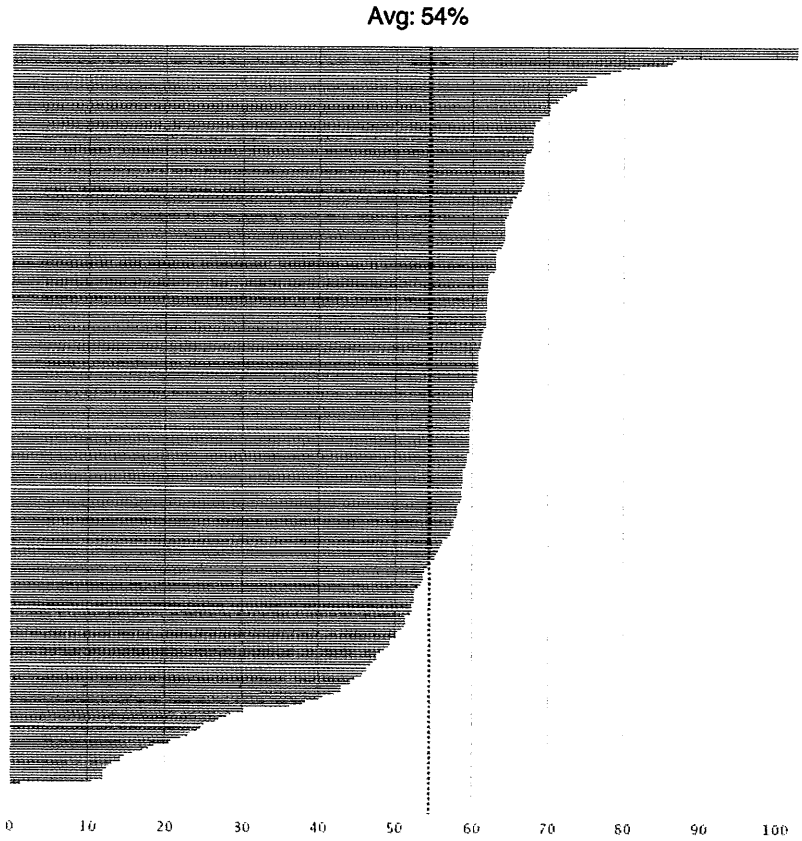
Howard County Public Schools has a variety of administrator roles at both the school and district level (e.g., instructional team leaders, resource teachers, and instructional facilitators) that could be utilized to help implement the shift in how special education staff use their time and serve students.

5a. Staff Time with Students

Special Education Teachers (Inclusion)

To the extent that special education teachers will be providing support in core academic subjects, there is an opportunity to have them spend more time doing so. In the current scheduling, special education teachers spend, on average, 54% of their time working directly with students. As a point of comparison, a general education teacher might typically spend 75%-85% of their week providing direct service and in some districts special education teachers also spend 75% of their time with students. Re-thinking the schedule and non-teaching demands of special educators in the district could allow the teachers to spend more of their week helping students.

Special Education Teacher (Inclusion) Direct Service (396 FTE)



- Special education teachers in Howard County Public Schools spend, on average, about 2.5 days per week with students.

All activities are important, yet few districts actively manage the distribution of this time for special education staff. For comparison, in general education all trade-offs between student time and indirect time are set by the district leadership, such as the number of courses taught by a high school math teacher.

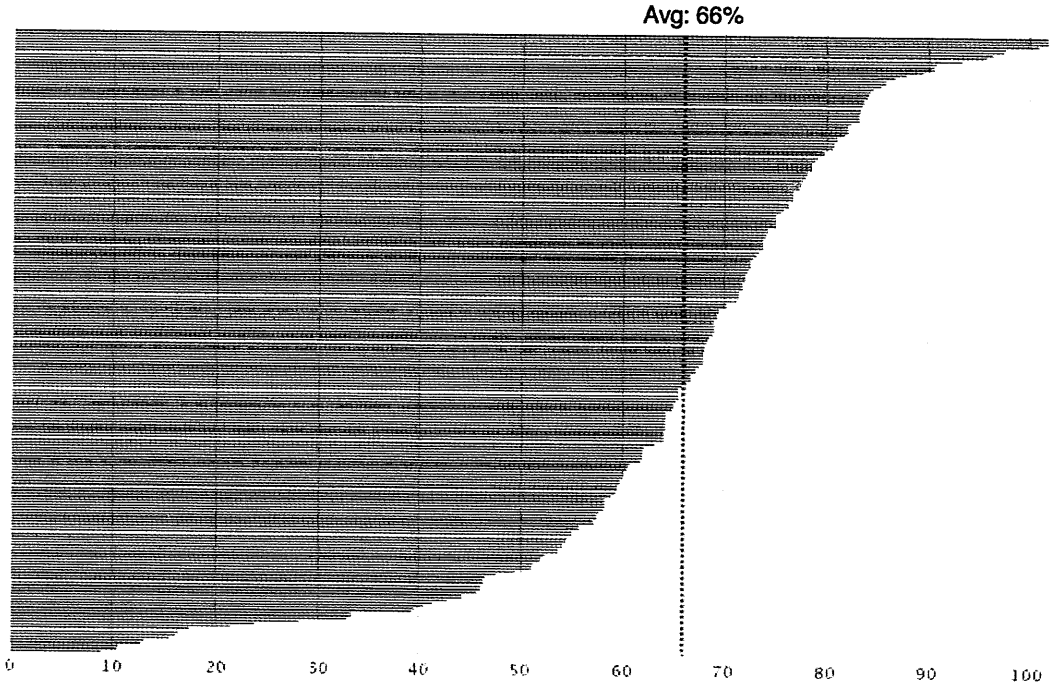
Special Education Teacher (Inclusion) Activities (396 FTE)

Activity	% time spent
Student instruction or support	54%
Total direct service	**54%**

Planning/ materials preparation	14%
Collaboration with colleagues (email, phone, in-person)	7%
Paperwork/ IEP writing	6%
Personal lunch	6%
Attend school based meeting (other than IEP)	4%
Assigned school duties (i.e. bus duty, lunch duty, etc.)	3%
Parent communication (email, phone, in-person)	2%
Student observation/ data collection	2%
Attend meeting (IEP)	2%
Professional development	<1%
Scheduling	<1%
Implementation of specialized methodologies	<1%
IEP testing/ assessment	<1%
Travel	<1%
Over reported	-3%
Total Indirect Service	**46%**

- Special education teachers spend about 2.5 days per week with students and about one day per week planning or collaborating with colleagues.

Special Education Paraeducator (Inclusion) Direct Service (212 FTE)



- Special education paraeducators in Howard County Public Schools provide slightly more than three days per week of direct service to students.

Special Education Paraeducator (Inclusion) Activities (212 FTE)

Activity	% time spent
Student instruction or support	66%
Total direct service	**66%**

Planning/ materials preparation	9%
Assigned school duties (i.e. bus duty, lunch duty, etc.)	7%
Personal lunch	7%
Student transition/ escort	2%
Attend meeting	1%
Data collection	1%
Behavior intervention plan	<1%
Parent communication (email, phone, in-person)	<1%
Implementation of specialized methodologies	<1%
Travel	<1%
Under reported	5%
Total Indirect Service	**34%**

5b. Implementing the Shift

Implementing changes such as shifting staff schedules to spend more time with students will require focus and effort on the part of the district. To accomplish this, it can help to have some staff in the district have the explicit role of supporting the scheduling and managing the daily activities of special education staff.

In Howard County Public Schools there are a variety of special education administrators, both at the school and district level that could fill this role, including resource teachers, instructional facilitators, and secondary instructional team leaders. Interviews indicated that oftentimes the roles and responsibilities of each of these three roles are ambiguous or overlapping. This is an opportunity to more explicitly define the expectations for each position. While redefining these roles, the district could include a new expectation that some will manage the schedules and service delivery model.

REPORT

HOWARD COUNTY
PUBLIC SCHOOL SYSTEM

BOARD OF EDUCATION OF HOWARD COUNTY
MEETING AGENDA ITEM

TITLE: PreK-12 Mathematics Program **DATE:** March 24, 2016

PRESENTER(S): John SanGiovanni, Coordinator, Elementary Mathematics; Jennifer Novak, Acting Coordinator, Secondary Mathematics; Ebony Langford-Brown, Executive Director, School Improvement and Curricular Programs

VISION 2018 GOAL: ☒ Students ☒ Staff ☒ Families and Community ☒ Organization

OVERVIEW:

The purpose of this report is to provide information about how one of our most critical programs, mathematics, has been enhanced to encompass national standards, better align the program to Vision 2018, and ensure that all students graduate college and career ready. This will be accomplished through the three components of the PreK-12 Mathematics Instructional Plan: rigorous first instruction, high quality differentiated support, and increased access.

RECOMMENDATION/FUTURE DIRECTION:

In every mathematics classroom, ensure that teachers engage students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and to reason mathematically. Continue to consistently increase the percentage of students, from all student groups, who successfully complete two years of college-level mathematics prior to graduation.

SUBMITTED BY:

Ebony Langford-Brown,
Executive Director, School Improvement and Curricular Programs

Caroline Y. Walker,Director,
Curricular Programs,
Elementary and PreK-12

William J, Barnes, Director,
Curricular Programs,
Secondary and PreK-12

APPROVAL/CONCURRENCE:

Renee A. Foose, Ed.D.
Superintendent

Linda T. Wise,
Deputy Superintendent

Introduction

The Howard County Public School System (HCPSS) seeks to ensure that every student achieves academic excellence in an inspiring, engaging, and supportive environment. In every mathematics classroom, teachers must engage students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and to reason mathematically (NCTM, 2014). To achieve this, there must be effective, consistent, and impactful implementation of the Maryland College and Career-Ready Standards for Mathematics (MCCRS-M), with fidelity, and at the intended level of rigor. The purpose of this report is to provide information about how one of our most critical programs, mathematics, has been enhanced through a program plan process to encompass these standards, better align the program to *Vision 2018*, and ensure that all students graduate college and career ready.

Rationale

Some HCPSS students, especially those from historically underserved populations and/or those receiving special services (i.e., those who have IEPs, are English language learners, or are eligible for Free and Reduced-Price Meals (FARMs)), are not attaining the highest levels of mathematics learning. In the Howard County Public School System, "effective teaching is the non-negotiable core that ensures that all students learn mathematics at high levels and...such teaching requires a range of actions at the district, school and classroom levels" (NCTM, 2014, p. 4). Access and equity are the cornerstones of a robust mathematics program. A responsive mathematics program provides opportunities for high-level mathematics instruction and supports when needed and considers the experiences students bring with them. HCPSS's goal is to consistently increase the percentage of students, from all student groups, who successfully complete two years of college-level mathematics prior to graduation.

The HCPSS has developed research-informed strategies to ensure that each and every student has an opportunity to access rigorous, college-level, mathematics courses prior to graduation. Implementation of the HCPSS Mathematics Program with fidelity is crucial in realizing the goals set forth in *Vision 2018* for students in Prekindergarten through Grade 12. The following outcomes are particularly relevant:

- Outcome 1.1 - The instructional program is rigorous, globally-relevant, and aligned with international and/or nationally recognized college and career readiness standards.
- Outcome 1.2 - Students have equitable access to a rigorous instructional program.
- Outcome 1.3 - Technology is leveraged so that students have access to learning experiences that meet their needs and interests.
- Outcome 1.4 - Students are engaged in the learning process.
- Outcome 1.5 - Students meet or exceed rigorous performance standards

- Outcome 1.6 - Meaningful measures of student outcomes are in place.
- Outcome 2.2 - Staff members have access to learning experiences that support their professional growth.
- Outcome 2.3 - Staff members are held accountable for and supported in meeting standards-based performance expectations.
- Outcome 3.3 - HCPSS engages families and the community through relevant, timely, accessible, and audience-focused communications.
- Outcome 4.2 - HCPSS hires and retains a talented, effective, and diverse workforce.
- Outcome 4.6 - Decisions are informed by relevant data in all operational areas.

It is important to communicate consistent messages when discussing mathematics instruction with all HCPSS stakeholders. To that end, creating an instructional plan for PreK-12 mathematics becomes essential for establishing consistency and quality. Creating shared goals and actions will connect the varied stakeholders involved in successful implementation of the mathematics program.

The PreK-12 Mathematics Instructional Plan

There are three primary components for the PreK-12 Mathematics Instructional Plan. The components are rigorous first instruction, high quality differentiated support, and increased access.

First Instruction

First (initial) instruction must be rigorous, aligned with evidence-based strategies, and implemented by highly-skilled mathematics teachers. All students deserve to receive high quality first instruction in mathematics in a clearly defined and structured setting to develop a solid foundation for substantial growth in knowledge and skills. Such a setting includes refined expectations for the math instructional block.

High-quality professional learning opportunities need to be provided to school-based instruction personnel. A comprehensive professional learning approach would include scheduling expectations for common planning time within teams during the Program Implementation Period (PIP), providing structure to PIP time, encouraging teamwork at the high school level, and building capacity of the mathematics team leaders (Mathematics Support Teachers (MSTs/MISTs), Teacher Development Liaisons (TDLs), etc.) It would also include the use of online elements.

Curriculum leaders assist teachers by defining, implementing, and monitoring expectations for high quality first instruction (fidelity of implementation). These expectations need to be reflected in standards-based grading and reporting, consistent processes for the use of assessment data,

and a communication model that engages all stakeholders (e.g., an HCPSS-produced math support website for students and their families).

Pedagogy

First instruction should feature effective, evidence-based pedagogy. In 2014, the National Council of Teachers of Mathematics (NCTM) published *Principles to Action, Ensuring Mathematical Success for All*. This work is based upon classroom best practices and evidence from research on mathematics programs. *Principles to Actions* establishes guidance on what instructional practices and supporting elements are critical to overcoming the obstacles that currently prevent mathematical success for all students. The eight "Mathematics Teaching Practices" represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics. These eight practices are:

1. **Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
2. **Implement tasks that promote reasoning and problem solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
3. **Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
4. **Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
5. **Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
6. **Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly to solve contextual and mathematical problems.
7. **Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
8. **Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to continually adjust instruction in ways that support and extend learning.

This framework offers all HCPSS educators a common lens for collectively moving toward improved instructional practice and for supporting one another in becoming skilled at teaching in ways that matter to ensure successful mathematics learning for all students. See Appendices A and B for the Elementary and Secondary Lookfor Documents.

Content

Exemplary initial instruction must be rigorous and balanced. Balanced mathematics instruction pursues deep learning through conceptual understanding, procedural fluency, and application of mathematics to "real world" situations. For years, rote, procedural learning has been the norm in mathematics. This type of learning is ineffective for many students, especially those who have trouble understanding the larger mathematics concepts behind mathematical practices. In *Adding It Up: Helping Children Learn Mathematics*, The National Research Council (2001) described five strands of mathematical proficiency:

1. *Conceptual understanding*—comprehension of mathematical concepts, operations, and relations
2. *Procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
3. *Strategic competence*—ability to formulate, represent, and solve mathematical problems
4. *Adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification; and
5. *Productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Building from those proficiencies, the Council of Chief State School Officers (CCSSO, 2013) has identified specific mathematics instructional shifts, with *rigor* defined as an "equal intensity of conceptual understanding, procedural skill and fluency, and applications."

- **Conceptual understanding**: comprehension of mathematical concepts, operations, and relations
- **Procedural Skill and Fluency:** skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Application:** application of mathematics understanding to solve real-world problems and connect to other mathematical concepts (e.g., the distributive property can be applied to find the area of an irregular polygon)

The PreK-12 Mathematics content standards convey a unified vision of the mathematical big ideas and reflect a progression of learning that is meaningful (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). These learning

progressions are organized by the following multi-grade content domains or conceptual categories:

- Counting and Cardinality (PreK-K)
- Operations and Algebraic Thinking (PreK-5)
- Number and Operations in Base Ten (K-5)
- Measurement and Data (PreK-5)
- Geometry (PreK-HS)
- Numbers and Operations--Fractions (3-5)
- The Number System/Number and Quantity (6-HS)
- Ratios and Proportional Relationships (6-7)
- Expressions and Equations (6-8)
- Functions (8-HS)
- Statistics and Probability (6-HS)
- Algebra (HS)

The Standards for Mathematical Practice (SMP) (NGA & CCSSO, 2010) describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on those "processes and proficiencies" that have had longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). Multiple practices are evident in exemplary initial instruction. These Standards include:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Rigor is another component that is reflected in the cognitive demand of the tasks used in the mathematics classroom. Webb's Depth of Knowledge (DOK) model is employed to analyze the cognitive expectation demanded by standards, curricular activities, and assessment tasks (Webb,

1997). Initial mathematics instruction must balance the levels of Depth of Knowledge. The Depth of Knowledge levels are:

- Level 1 - Recall and Reproduction
- Level 2 - Skills and Concepts
- Level 3 - Short-Term Strategic Thinking
- Level 4 - Extended Thinking

Mathematics Instructional Time

At the elementary level, consistent instructional models for the 75-minute mathematics block must be implemented. Mathematically proficient students are not developed through lecture-based, whole-group instruction. Mathematics instruction must be engaging and differentiated to meet the needs of diverse learners. The 75-minute mathematics block must include time for engaging differentiated instruction.

Number concepts are the foundation of elementary mathematics. These concepts are complex and interconnected. Fluency with number concepts and computation must be developed over the course of a full school year. Every mathematics class should begin with a number routine to develop these ideas.

Closure of each lesson is critical for student learning. Closure is an opportunity for students to share their ideas and observations. It is an opportunity for teachers to connect the concepts presented in the instructional activities to real-world examples. Closure can feature class discussion or independent journal activities. It can also be an opportunity for gathering formative assessment data. Every mathematics class should end with closure.

Instructional time must be maximized by minimizing tasks with little or no instructional value. Procedures for station rotations, material distribution, or homework review/grading must be streamlined and efficient. Appendix C describes the elementary instructional models in more detail.

At the secondary level, instructional models for support extend beyond the daily 50-minute mathematics block. The 50-minutes of initial instruction embed the practices described earlier. These include the use of effective formative assessment techniques that provide evidence for supporting students during classroom instruction. Differentiated approaches addressing specific student needs are an expected component of high quality mathematics instruction. Selection and implementation of worthwhile mathematical tasks provide students multiple access points to demonstrate understanding. Appendix D describes the secondary instructional models in more detail.